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Paths to victory in presidential elections: the setup power of noncompetitive states

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Abstract In US presidential elections, voters in noncompetitive states seem not to count—and have zero power, according to standard measures of voting power—because they cannot influence the outcome in their states. But the electoral votes of these states are essential to a candidate's victory, so they do count, but in a different way. We propose a simple model that enables us to measure the setup power of voters in noncompetitive states by modeling how these states structure the contest in the competitive states, as illustrated by the 2012, 2008, 2004, and 2000 presidential elections. We define three measures of setup power—winningness, vulnerability, and fragility—and show how they pinpoint the advantages of the candidate who leads in electoral votes of noncompetitive states. In fact, this candidate won in all four elections.

Keywords Presidential elections · Electoral College · Measure of voting power · Setup power of noncompetitive states

1 Introduction

The role of the Electoral College in US presidential elections has been controversial since the Constitution was adopted in 1789. It became more so starting in the 1830s, when states began to require that all of their electoral votes be cast in favor of the state's plurality-vote winner, as remains true today in all states except Maine and Nebraska. The controversy was exacerbated by the 2000 election when, for the third time—the two previous instances

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were in 1876 and 1888—the popular-vote winner (Al Gore) was not the electoral-vote winner (George Bush).[1]

The winner-take-all rule not only has produced divided verdicts, as in 2000, but also has had a profound effect on how candidates campaign in presidential elections. Especially in recent elections, candidates have spent almost all their campaign resources in the competitive states, defined as the states with the tightest races, based on state polls.[2] Additional expenditures in these states may well make the difference between victory and defeat—both in the state and nationally.

In this paper, we argue that the noncompetitive states are just as important to the outcome as the competitive states, but in a different way. Noncompetitive states do not have a direct impact on which candidate wins or loses on election day; instead, they set up the contest between the two major-party candidates, an observation that remains true even if another candidate wins some electoral votes in noncompetitive states.[3]

We call the ability of the noncompetitive states to help a candidate their *setup power*. The outcomes in these states, based on state polls, are not in doubt, giving an edge to the candidate leading before the election.

In 2012, for example, a consensus among pollsters and analysts was that ten states were "up for grabs," though in the actual election Mitt Romney won only one of these states (North Carolina). During the campaign, both Barack Obama and Romney made multiple campaign appearances in all ten of these states, virtually ignoring the other 40 states and the District of Columbia—except for occasional fundraising visits (especially in California and New York). Clearly, the candidates felt that that none of the noncompetitive states could be wrested from the candidate who was substantially ahead in the polls.

A consensus about which states are competitive in an election does not always exist. We use a *retrospective criterion*—that the winner's margin of victory in the actual election be less than or equal to 6 % (we suggest a modification in Sect. 5)—to distinguish competitive from noncompetitive states. This margin comports with the usual margin of error of ± 3 % in most state polls: Polls that predict a win by more than 6 % indicate that a state is almost surely not competitive.

In 2012, our retrospective criterion identifies 8 of the 10 states that were judged competitive by pollsters and in which the candidates campaigned. Our criterion excludes Nevada and Wisconsin, which Obama won by 6.7 and 6.9 %, respectively; these two states, it turns out, were the next-most competitive in the election, attesting to the accuracy of polling in 2012. The close agreement of our criterion with perceptions of competitiveness before the election, based on state polls, suggests its validity as an indicator of which states—admittedly ex post—could have made a difference in the outcome.

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1 In another election (1824), no majority electoral-vote winner emerged, so the election was decided in the House of Representatives. However, because electors in six states were appointed legislatively rather than elected popularly, one cannot determine who the popular-vote winner was (Andrew Jackson received a plurality of electoral votes but lost the election to John Quincy Adams in the House).

2 Fairvote.org (2016) reported, based on data compiled by the *Washington Post*, that in the 2012 presidential election, ten states it identified as "battleground" (or competitive) received 99.6 % of the money spent on advertising by the major-party campaigns and their allies. For an even more dramatic comparison, consider two adjacent New England states, New Hampshire (a battleground state) and Vermont (a Democratic state). National Public Radio (2016) reported that, in 2012, New Hampshire received 1000 times more presidential campaign spending per capita than Vermont.

3 The last presidential election in which a third-party candidate won any electoral votes was 1968, when George Wallace, who garnered 13.5 % of the national popular vote, received a plurality of popular votes in five southern states and, therefore, all of their electoral votes. In 1992, Ross Perot won a larger proportion of the national popular vote (18.9 %) but failed to win any states or electoral votes.

To single out competitive states as critical in an election ignores the fact that electoral votes from noncompetitive states, even though they effectively have been decided before the election, are just as critical to a candidate's victory as electoral votes from competitive states. While individual voters in noncompetitive states could not have changed the outcome by switching their votes, they, nevertheless, influence the outcome.[4] By setting up the contests in the competitive states, they have a kind of agenda control. But how best to measure this setup power is less clear.

In fact, we will propose three different measures of this power in order to assess its effects in the last four US presidential elections. But before comparing these elections, we look at the 2012 race to illustrate our findings—in particular, how an uneven distribution of electoral votes in the noncompetitive states facilitated Obama's victory.

2 The 2012 election

As we noted earlier, eight states, which were won by margins of 6 % or less, were competitive in 2012 by our retrospective criterion. Those states collectively accounted for 114 of the available 538 electoral votes. Because Barack Obama had a 233–191 electoral-vote lead over Mitt Romney in the 42 noncompetitive states and the District of Columbia, he needed only 37 of the 114 electoral votes in the competitive states to win with a majority of 270 electoral votes, whereas Romney needed 79.

Each of the eight competitive states can support either Obama or Romney, so altogether the states could have split in $2^8 = 256$ ways.[5] In each split, one or other of the candidates becomes the national electoral-vote winner (or, possibly, the election ends in a tie, to be decided by the House of Representatives).

We measure a candidate's *winningness* as the proportion of these splits in which he or she is the winner. Note that each candidate's electoral votes come from noncompetitive states—effectively decided before the election—and the competitive states that support him or her on election day (we discuss the treatment of ties later). This calculation showed that 207 (80.9 %) of the 256 splits would result in a win for Obama, whereas only 49 (19.1 %) would result in a win for Romney, giving Obama 4.22 times more ways of winning than Romney.

Additionally, in 114 of Obama's 207 winning coalitions (55.1 %), no competitive state—even the largest, Florida—could change the outcome by switching to the other candidate, whereas in 46 of Romney's 49 winning coalitions (93.9 %), at least one state could defect and rob him of victory, making Romney's winning coalitions much more vulnerable. We measure a candidate's *vulnerability* as the proportion of his or her winning coalitions of competitive states in which a single competitive state, by switching to the other candidate, either can cause a change in the winner or create a tie. A candidate's

[4] Standard measures of voting power, such as the Shapley–Shubik index (1954) or the Banzhaf index (1965, 1968), would say that voters in these states have no voting power because they will never be pivotal—they will always vote for one party. For comprehensive information on measures of voting power, see Felsenthal and Machover (1998) and Holler and Nurmi (2013). The focus of these measures is on minimal winning coalitions, from which the defection of a player would cause the coalition to be losing, whereas our measures take into account all winning coalitions.

[5] As we show in Sect. 4, in 2004 and 2000 there were 12 and 16 competitive states, respectively, producing a dramatic increase in the number of possible splits to 4096 and 65,536.

fragility is measured by the expected number of competitive states in a winning coalition that can disrupt victory in this way.

By these measures, Romney's winning coalitions were 2.09 times more vulnerable, and 4.22 times more fragile, than Obama's, which is entirely attributable to the lead that Obama enjoyed in the noncompetitive states. In sum, the noncompetitive states count, but they exercise power differently from the competitive states.

The paper proceeds as follows. In Sect. 3, we give a general description of our model of a presidential election, illustrating it with a hypothetical example in which there are five competitive states and, therefore, $2^5 = 32$ possible splits. We list these splits and show explicitly the leading candidate's advantages—in greater opportunities to win, and in diminished vulnerability and fragility—when he or she needs fewer electoral votes from the competitive states than his or her opponent to win the contest.[6]

In Sect. 4, we report the results of these calculations for the competitive states in 2012, 2008, 2004, and 2000. In those years, there were 8, 7, 12, and 16 competitive states, respectively, by our retrospective criterion, making complete listings of all splits impractical. These elections include two wins by a Democrat, Barack Obama, in 2012 and 2008, and two wins by a Republican, George W. Bush, in 2004 and 2000. They illustrate two extremes—(i) Bush needed every one of the 4 (of 16) competitive states that he actually won in 2000 to secure victory, whereas (ii) Obama needed none of seven competitive states to win in 2008 (he won 4)—and two more conventional races, in which each candidate needed to win some but not all of the competitive states to emerge victorious.

In Sect. 5, we ask whether the data on election outcomes are consistent with the assumptions of our probabilistic model. The preponderance of competitive states favoring one candidate in 2000 and 2012 casts some doubt on the assumption that these states made independent, equiprobable choices, but if the 6 % criterion for measuring competitiveness is halved to 3 %, the observations are entirely consistent with the assumption of independence.

In Sect. 6 we draw several conclusions. In particular, we argue that the noncompetitive states, far from being ciphers, exercise power indirectly in a way that differs from that of the competitive states.

3 Model and example

Denote the two presidential candidates by X and Y. We first describe the probabilistic election model that essentially underlies the calculation of the Penrose–Banzhaf–Coleman measure of voting power, which we call the *Banzhaf model*. Suppose that the contest involves n states, numbered $i = 1, 2,\ldots, n$, and that state i has v_i electoral votes. Let the random variable V_i equal 1 if state i votes for candidate X and 0 if state i votes for candidate Y. The number of electoral votes for candidate X is $V = \sum_{i=1}^{n} V_i$.

In the Banzhaf model, each V_i is a Bernoulli random variable (i.e., binomial, with 1 trial) that equals 0 or 1 with parameter $p = ½$, mimicking the toss of a fair coin. Moreover, the election outcomes for all states (coin tosses) are independent. It follows from the linearity of V that the expected number of electoral votes for candidate X is $e/2$, where $e = \sum_{i=1}^{n} v_i$ is the total number of electoral votes across all states.

[6] Our measures are related to two measures proposed in Coleman (1971)—the "power to initiate action" and the "power to prevent action"—where the action in our case is winning the election. Our three measures, unlike Coleman's, are tailored specifically to capture the effects of winner-take-all in the Electoral College; they cannot be derived from Coleman's.

Of course, most states are not equally likely to support one or the other of the candidates. In our model, we assume that all states are either (i) competitive or (ii) noncompetitive, in which case they are committed to support either candidate X or candidate Y. For convenience, let states 1, 2,…, m be the competitive states, where $m < n$. Let $V' = \sum_{i=1}^{m} V_i'$ be the total number of electoral votes for candidate X in these states, and let $e' = \sum_{i=1}^{m} v_i'$ be the total number of their electoral votes.

The net effect of the noncompetitive states is that there exist thresholds, t_X and t_Y, such that X wins if $V' > t_X$ and Y wins if $V' < t_Y$. Assuming that no third party receives any electoral votes, then either $t_X = t_Y - 1$ or $t_X = t_Y$. To illustrate, if the noncompetitive states split evenly, and the competitive states have 5 electoral votes, $t_X = 2$ and $t_Y = 3$, because X wins by getting more than 2 electoral votes, and Y wins if X gets less than 3. But if the competitive states have 6 electoral votes, $t_X = t_Y = 3$, because in a 3-3 tie, neither candidate wins.

In 2012, $m = 8$ and $e' = 114$. Obama would win the election if he obtained at least 37 electoral votes in the competitive states, and Romney would win the election if he obtained at least 79. If Obama is candidate X, $t_X = t_Y = 36$; if Romney is, $t_X = t_Y = 78$. If the candidates had split the 424 electoral votes in the noncompetitive states evenly (212 each), then each candidate would have needed at least 58 of the 114 electoral votes in the competitive states to win, making $t_X = t_Y = 57$. In this case, a tie would occur if each candidate receives $212 + 57 = 269$ electoral votes, which can occur in several ways.

We henceforth apply the equiprobability assumption to just the m competitive states, so for $i = 1, 2, \ldots, m$, each V_i' is a Bernoulli random variable (binomial, with 1 trial) that equals 0 or 1, with parameter $p = ½$. This yields a *split*, or division of the competitive states, into supporters of X (with $V_i' = 1$) and supporters of Y (with $V_i' = 0$). If we further assume that $V_1', V_2' \ldots, V_m'$ are independent, each possible split is equally probable.

Counting the splits of the competitive states is a way of measuring the probability that X wins ($V_i' > t_X$) or Y wins ($V_i' < t_Y$). For example, if there are exactly x splits in which candidate X wins, then the probability of a win by X is $x/2^m$. By counting the number of splits that lead to every possible outcome, we can determine the "winningness" of each candidate as well as the vulnerability and fragility of his or her winning coalitions.

We assume that outcomes in the noncompetitive states are determined prior to the election. By contrast, because the competitive states are viewed as equally likely to vote for X or Y, we model their outcomes as coin tosses, with $p = ½$. The assumption that $V_1', V_2' \ldots, V_m'$ are independent may be more problematic; we assess its validity later.

Our model does *not* say that every voter in a competitive state is equally likely to vote for X or Y. In fact, most voters in competitive states are *partisan* and decide well before an election how they will vote. At the time of the election, a relatively few undecided voters in a competitive state, whom we call *nonpartisan*, may be thought of as deciding whether the state will vote Democratic or Republican. If the voters, both partisan and nonpartisan, split approximately evenly between the Democratic and Republican candidates in each competitive state, these states become, in the parlance of electoral politics, "too close to call."[7]

We now illustrate our model with a simple hypothetical example. Assume that there are two candidates, X and Y, and 5 competitive states (A, B, C, D, E), which have, respectively, (2, 3, 4, 5, 6) electoral votes (total: 20). The winner in each state wins all of the electoral votes of that state.

[7] An exactly even split of partisan voters between the two parties will almost never occur, so it is more accurate to say that the split is within some margin of error of being 50–50.

We assume that X has a substantial lead over Y in the noncompetitive states, needing only 6 of the 20 electoral votes to win, whereas Y needs 15. In our earlier notation, $t_X = 5$ and $t_Y = 6$. In Table 1, we list the 32 possible *splits* (i.e., divisions into competing coalitions) of the states, from X winning all five states to Y winning all five.

For each of the 32 splits of the five competitive states, we have indicated with an asterisk in Table 1 which state(s) are *critical* to the winning coalitions: Whenever such a state changes its vote, the previously winning candidate would lose to (or tie) his or her opponent, which constitutes a *critical defection*. Notice that in every split, either X wins with at least six electoral votes, or X receives five or fewer electoral votes, in which case Y wins with at least 15. In this example, a tie is not possible.

But as we will show later, if each candidate could have won with a simple majority of 11 of the 20 electoral votes, then a 10–10 split, in which both candidates tie, is possible. However, we will not treat ties as losing but, instead, as halfway between winning and losing, in a manner to be made precise later.[8]

For an integer n satisfying $0 \leq n \leq 32$, define a candidate's *winningness index*, $W(n)$, to be the percentage of the 32 splits in which he or she obtains at least n electoral votes. Table 1 shows that when X needs at least 6 votes, and Y at least 15, to win, then

$$W(6) = 26/32 = 81.25\ \%; \quad W(15) = 6/32 = 18.75\ \%.$$

Because X wins in 26 splits and Y in 6, X has 26/6 = 4 1/3 times more opportunities to win. Observe that $W(6) + W(15) = 100\ \%$, reflecting that ties are not possible in this example.

Assume a candidate needs at least n electoral votes to win. Then a candidate's *vulnerability index*, $V(n)$, is the (conditional) probability that his or her support includes at least one critical state, given that he or she wins. Table 1 shows that 13 of X's 26 winning coalitions, and all 6 of Y's 6 winning coalitions, include a critical state, so

$$V(6) = 13/26 = \tfrac{1}{2} = 50\ \%; \quad V(15) = 6/6 = 100\ \%.$$

Because these conditional probabilities reflect different scales (note that they do not have a common denominator), their sum is not fixed, though they both have an upper bound of 100 % (i.e., at most all of a candidate's winning coalitions can be vulnerable). The percentage gap in our example is $V(15) - V(6) = 50\ \%$; in ratio terms, as seems more intuitive, Y's winning coalitions are twice as vulnerable as X's.

$V(n)$ does not take into account the number of members of a winning coalition that render it vulnerable. Another useful measure is a candidate's *fragility index*, $F(n)$, or the expected the number of critical states in a winning coalition. From Table 1 we can calculate this expected value for each candidate by adding, for each of his or her winning coalitions, the number of states that could make a defection critical, and dividing this result by the number of winning coalitions of each candidate:

$$F(6) = [13(0)\ +\ 8(1)\ +\ 5(2)]/26 = 18/26 \approx 0.69;$$

$$F(15) = [1(1) + 3(3) + 2(4)]/6 = 18/6 = 3.$$

8 In the case of a tie in the Electoral College, or if no candidate wins a majority of electoral votes because of a split among more than two candidates, the election is decided by a simple majority in the House of Representatives, wherein each state has one vote, as was formalized by the 12th Amendment (1804).

Table 1 Thirty-two splits and critical states in *X's* and *Y*'s winning coalitions, in a 5-state example wherein *X* needs 6 and *Y* needs 15 electoral votes to win

Split	X	Y	Winner	# States in winning coalition	# States critical
1	ABCDE		X	5	0
2	BCDE	A	X	4	0
3	ACDE	B	X	4	0
4	ABDE	C	X	4	0
5	ABCE	D	X	4	0
6	ABCD	E	X	4	0
7	CDE	AB	X	3	0
8	BDE	AC	X	3	0
9	BCE	AD	X	3	0
10	BCD	AE	X	3	0
11	ADE	BC	X	3	0
12	ACE	BD	X	3	0
13	ACD	BE	X	3	0
14	ABE*	CD	X	3	1
15	ABD*	CE	X	3	1
16	ABC*	DE	X	3	1
17	DE*	ABC	X	2	1
18	CE*	ABD	X	2	1
19	C*D*	ABE	X	2	2
20	BE*	ACD	X	2	1
21	B*D*	ACE	X	2	1
22	B*C*	ADE	X	2	2
23	AE*	BCD	X	2	1
24	A*D*	BCE	X	2	1
25	A*C*	BDE	X	2	2
26	AB	C*D*E*	Y	3	3
27	E*	ABCD	X	1	1
28	D	A*B*C*E*	Y	4	4
29	C	A*B*D*E*	Y	4	4
30	B	AC*D*E*	Y	4	3
31	A	BC*D*E*	Y	4	3
32		ABCDE*	Y	5	1

A defection by a state with an asterisk in one of *X*'s or *Y*'s winning coalitions causes it to lose. Thirteen of *X*'s 26 winning coalitions (50 %), and 6 of *Y*'s 6 winning coalitions (100 %), are vulnerable, which are the values of $V(6)$ and $V(15)$. *X*'s winning coalitions have an average of $16/26 \approx 0.69$ critical states, and *Y*'s have an average of $18/6 = 3$ critical states, which are the values of $F(6)$ and $F(15)$

In ratio terms, *Y*'s winning coalitions are $(18/6)/(18/26) = 26/6 = 4\ 1/3$ times as fragile as *X*'s.

It is no accident that the ratio of *X*'s winningness to *Y*'s, and *Y*'s fragility to *X*'s, are equal,

$$\frac{F(6)}{F(15)} = \frac{W(15)}{W(6)} = 4\ 1/3,$$

even though winningness measures the proportion of winning coalitions and fragility measures the expected number of coalition members that render a winning coalition vulnerable. Because any defection that changes *Y* from winning to losing can be reversed to create a defection that changes *X* from winning to losing, and vice versa, both *X* and *Y* face the same number of possible defections, making the numerators of *F*(6) and *F*(15) the same (18 in our example). The denominators of *F*(6) and *F*(15), namely *W*(6) and *W*(15), are the numbers of winning coalitions of *X* and *Y*, which are inverted in the quotient shown above.

Although these ratios convey the same information about *X*'s advantage over *Y*, the actual values of *W*(*n*) and *F*(*n*) describe different aspects of *X*'s advantage. In particular, *X*'s winningness (81.25 %) does not shed light directly on *X*'s robustness—on average only 0.69 of the five states (13.8 %) render *X*'s coalitions vulnerable, whereas on average three states (60 %) render *Y*'s coalitions vulnerable. And neither winningness nor fragillty directly illuminate *X*'s lack of vulnerability—that only half of *X*'s 26 winning coalitions are vulnerable, whereas all of *Y*'s six winning coalitions are vulnerable.

In sum, our three indices measure the Electoral College strength of a candidate in different ways, where "strength" is increasing in *W*(*n*) and decreasing in *V*(*n*) and *F*(*n*). But what if the competitive states had split evenly so that, instead of the thresholds of 6 and 15 electoral votes, each candidate needed a simple majority of at least 11 of the 20 electoral votes to win? While *X* and *Y* will have the same index values, it is useful compare them with their previous values when there was not an even split of electoral votes in the noncompetitive states.

For this purpose, we show in Table 2, where we assume that *X* is the winning candidate, the 16 splits in which each candidate wins with at least 11 electoral votes, or ties with 10, of the 20. We call this situation the *baseline*.

In two of the 17 splits in Table 2 (#7 and #16), no winner emerges because there is a 10–10 tie between *X* and *Y*. In this split, all five states are critical, because any state that defects from either *X* or *Y* causes the candidate's tied coalition to lose.

We count a critical defection of a state from a tied coalition of *X* (#7 and #16 in Table 2) as ½ of a defection from a winning coalition, based on the assumption that a tie is worth only ½ of a win (i.e., a tie is equally likely to be broken in favor of either candidate). By the same token, if a state's defection causes a winning coalition to tie with the other coalition (as is possible in splits #4, #6, #8, #10, and #13), it also counts as half a defection, because the winning coalition does not lose, although it is prevented from winning.

In sum, either making or breaking a tie through a defection has 50 % of the value of defection from a winning coalition that causes it to lose. Using the formulas given previously for winningness, vulnerability, and fragility, the baselines values when *X* and *Y* need at least 11 electoral votes to win are as follows:

$$W(11) = 50\ \%;\ \ V(11) \approx 78.1\ \%;\ \ F(11) \approx 1.69.$$

In Table 3, we pull together the index values of winningness, vulnerability, and fragility—including the differences in these values and their ratios when $n = 6$ and $n = 15$—and show their percentage changes from the baseline figures given above. In the case of winningness, the change for *X* and *Y* from their baseline figure of 50 % is 31.25/50 = 62.5 %, up for *X* and down for *Y*.

Table 2 Sixteen splits (#7 and #16 are the same), and critical states in *X*'s winning and tied coalitions, in a 5-state example wherein *X* needs 11 electoral votes to win

Split	X	Y	Winner	# States in X's winning or tied coalitions	# States critical
1	*ABCDE*		*X*	5	0
2	*BCDE*	*A*	*X*	4	0
3	*ACDE*	*B*	*X*	4	0
4	*ABD(E)*	*C*	*X*	4	½
5	*ABCE**	*D*	*X*	4	1
6	*AB(C)D**	*E*	*X*	4	1½
7	*A*B*D**	*C*E**	Tie	3	1½
8	*C(D)E**	*AB*	*X*	3	1½
9	*BD*E**	*AC*	*X*	3	2
10	*(B)C*E**	*AD*	*X*	3	2½
11	*B*C*D**	*AE*	*X*	3	3
12	*AD*E**	*BC*	*X*	3	2
13	*(A)C*E**	*BD*	*X*	3	2½
14	*A*C*D**	*BE*	*X*	3	3
15	*A*B*E**	*CD*	*X*	3	3
16	*C*E**	*A*B*D**	Tie	2	1
17	*D*E**	*ABC*	*X*	2	2

A defection by a state with an asterisk in one of *X*'s winning or tied coalitions causes it to lose and is counted as 1, except in tied splits #7 and #16, where it counts as ½. A defection by a parenthesized player in splits #4, #6, #8, #10, and #13 causes *X* to tie with the *Y* coalition and is also counted as ½. Thirteen and one-half of *X*'s 16 winning and tied coalitions (78.1 %) are vulnerable to either kind of defection (#4 is half vulnerable), which is the value of $V(11)$; the 16 coalitions have an average of 27/16 $\approx$ 1.69 critical states, which is the value of $F(11)$

Table 3 Index values and percentage changes from baseline values, in a 5-state example

Index	X $N = 6$	Y $N = 15$	Absolute difference	Ratio
Winningness: $W(N)$ —Change from $W(11)$	81.25 % (+62.5 %)	18.75 % (–62.5 %)	62.5 %	4.33 (*X*/*Y*)
Vulnerability: $V(N)$ —Change from $V(11)$	50 % (–36.0 %)	100 % (+50 %)	50 %	2 (*Y*/*X*)
Fragility: $F(N)$ —Change from $F(11)$	0.69 (–59.2 %)	3 (+77.5 %)	1.77	4.33 (*Y*/*X*)

The baseline figure for vulnerability is 78.1 %, where the shift down for *X* to 50 % is 28.1/78.1 $\approx$ 36.0 %, and the shift up to 100 % for *Y* is 21.9/78.1 $\approx$ 28.0 %. The baseline figure for fragility is 1.69, and the shift down for *X* to 0.69 is 1/1.69 $\approx$ 59.2 %, and the shift up for *Y* to 3 is 1.31/1.69 $\approx$ 77.5 %.

To summarize, five of the six of the changes from the baselines, which are shown in parentheses in Table 3, are greater or equal to 50 %, with only the reduction in the vulnerability of *X* less (36.0 %). These shifts from the baselines, ranging from 36.0 % to

77.5 %, indicate that *X*'s lead in the noncompetitive states has a significant and sometimes dramatic effect on the candidates' political strength.

Only for winningness is the effect symmetric, helping *X* and hurting *Y* by the same amount. This index is probably *primus inter pares* as an indicator of the boost the leading candidate receives from being ahead in the noncompetitive states, although this is not to deny the importance of vulnerability and fragility as indicators of the robustness of the coalitions supporting *X*.

As noted in Sect. 1, we attribute *X*'s strength over what it would be if the playing field were level to the *setup power* of the noncompetitive states—their power to tilt the election in favor of *X*. In the extreme case in which the noncompetitive states already have decided the election, this power is total, because then none of the competitive states would be critical in any of the winning coalitions. As we will show in Sect. 4, this occurred in the 2008 presidential election.

4 Setup power in four US presidential elections

In Sect. 2, we previewed the results for the 2012 election, showing that Barack Obama was substantially favored over Mitt Romney according to our three measures of setup power—winningness, vulnerability, and fragility. In comparing this election to those of 2008, 2004, and 2000, we will not consider the baseline case, wherein the candidates are tied in electoral votes cast by the noncompetitive states, because this case never occurred in any of the four elections.

If it had occurred, each candidate would have had a 50–50 chance of winning, independent of k (i.e., $W(k) = ½$), where $k = e'/2 + 1$ if e' is even and $k = (e' + 1)/2$ if e' is odd. By contrast, $V(k)$ and $F(k)$ do depend on k: The larger is k, the more vulnerable and fragile are each candidate's winning coalitions, even though each candidate has the same probability of winning when they are tied in the noncompetitive states. Thereby the three indices cast different light on setup power, with vulnerability and fragility illuminating the stability, or lack thereof, of each candidate's winning coalitions, whereas winningness simply measures each candidate's probability of being victorious.

Consider two extreme cases when the winningness of both candidates is ½: (i) only one state is competitive and (ii) all states are competitive. In case (i), if the two candidates are tied in the noncompetitive states, the only winning coalitions for each candidate will be his or her noncompetitive states plus the competitive state, making their vulnerability and fragility equal to 1 (the one competitive state makes each winning coalition vulnerable, which gives it a fragility of 1). In case (ii), all *minimal winning coalitions*—from which at least one state's defection causes them to lose—will be vulnerable, and their fragility will equal the average number of states that all winning coalitions contain.

Because we have already discussed the considerable advantage that Obama enjoyed over Romney in 2012, based on his lead of 42 electoral votes in the noncompetitive states and the District of Columbia (see the summary statistics in Table 4), we turn next to the 2008 election, characterized by one fewer competitive state (7 in 2008 versus 8 in 2012).[9] Unlike 2012, however, Obama had a huge lead of $292 - 144 = 148$ electoral votes over his Republican opponent, John McCain, in the 43 noncompetitive states plus the District of Columbia.

[9] For all four presidential elections we analyze, we developed Excel spreadsheets to calculate our three measures of setup power, using both 6 and 3 % criteria to be competitive (see Sect. 5).

Table 4 Setup power in four US presidential elections

Election	Winningness	Vulnerability	Fragility
2012 Noncompetitive states— Obama led Romney 233–191; 8 competitive states (CO, FL, IO, NH, NC, OH, PA, VA) have 114 electoral votes 256 winning coalitions: Obama wins in 207 (93 vulnerable), Romney in 49 (46 vulnerable)	Obama: 0.809 Romney: 0.191 Ratio (O/R): 4.22	Obama: 0.449 Romney: 0.939 Ratio (R/O): 2.09	Obama: 0.850 Romney: 3.592 Ratio (R/O): 4.22
2008 Noncompetitive states—Obama led McCain 292–144; 7 competitive states (FL, GA, IN, MO, MT, NC, OH) have 102 electoral votes 128 winning coalitions: Obama wins in 128 (0 vulnerable), McCain in 0	Obama: 1.000 McCain: 0 Ratio (O/M): Infinite	Obama: 0 McCain: Undefined Ratio (M/O): Undefined	Obama: 0 McCain: Undefined Ratio (M/O): Undefined
2004 Noncompetitive states—Bush led Kerry 222–124; 12 competitive states (CO, FL, IO, MI, MN, NV, NH, NM, OH, OR, PA, WI) have 192 electoral votes 4053 winning coalitions (plus 43 tied): Bush wins in 3389 (1356 vulnerable), Kerry in 664 (610 vulnerable)	Bush: 0.833 Kerry: 0.167 Ratio (B/K): 4.98	Bush: 0.393 Kerry: 0.917 Ratio (K/B): 2.33	Bush: 0.892 Kerry: 4.441 Ratio (K/B): 4.98
2000 Noncompetitive states—Bush led Gore 211–181; 16 competitive states (AR, FL, IA, ME, MI, MN, MO, NV, NH, NM, OH, OR, PA, TN, WA, WI) have 146 electoral votes 65,005 winning coalitions (plus 531 tied): Bush wins in 57,104 (17,120 vulnerable), Gore in 7901 (6817 vulnerable)	Bush: 0.875 Gore: 0.125 Ratio (B/G): 7.03	Bush: 0.293 Gore: 0.861 Ratio (G/B): 2.94	Bush: 0.809 Gore: 5.684 Ratio (G/B): 7.03

The ties that can occur in 2004 and 2000 count for ½ a win in calculating winningness, vulnerability, and fragility (see Sect. 2 for details and an example)

In fact, Obama would have won the election by $292 - 246 = 46$ electoral votes even if he had lost all seven competitive states (with a total of 102 electoral votes), making his win a certainty. As it was, Obama won 4 of the 7 competitive states (Florida, Indiana, North Carolina and Ohio), with a total of 73 electoral votes, giving him an easy win of $365 - 173 = 192$ electoral votes in 2008. His national popular-vote margin over McCain was 7.27 % in 2008, compared with 3.86 % over Romney in 2012, which his electoral votes magnified, especially in 2008.

Because Obama had a certain win over McCain in 2008, his winningness is 1 and McCain's is 0, rendering the winningness ratio of the candidates (O/M) infinite. The fact that none of Obama's winning coalitions are vulnerable means that his vulnerability and fragility are 0, whereas those indices for McCain cannot be calculated because he has no winning or tied coalitions, no matter to which candidate the competitive states swing.

In 2004, George W. Bush led John Kerry in the noncompetitive states by a large margin of $222 - 124 = 98$ electoral votes. What tightened the election somewhat was that Kerry won in 7 of the 12 competitive states, obtaining 128 of their 192 electoral votes, but still enabling Bush to win by a comfortable $286 - 252 = 34$ electoral votes. Bush's national popular-vote margin was 2.52 %, slightly less than Obama's 3.86 % margin in 2012.

The presidential election of 2000 was the real squeaker, despite the fact that Bush had a lead of $211 - 181 = 30$ electoral votes over Al Gore in the 34 noncompetitive states and the District of Columbia. Because 16 states were competitive, however, and Gore won in 12 of them—picking up 85 of their 146 electoral votes—Bush in the end won by only 271

− 266 = 5 electoral votes, with one abstention. (If a "faithless elector" from the District of Columbia—which Gore won—had not abstained, Gore would have lost by 271 − 267 = 4 electoral votes.)

A switch of only two electoral votes would have created a 269–269 tie, which would have sent the election to the House of Representatives. The state with the closest popular-vote margin was Florida, which Bush won by a minuscule margin of 0.01 % (537 votes, after a Supreme Court decision), though Gore won the national popular vote by a margin of 0.52 % (543,995 popular votes).

The closeness of the 2000 election is not suggested by our indices of setup power: Bush had 7.03 times more winning coalitions than Gore, and Gore's winning coalitions were 2.94 times more vulnerable and 7.03 times more fragile than Bush's. These statistics raise the question of how probable it was that so many (75 %) of the 16 competitive states broke in favor of Gore in 2000, almost costing Bush that election. Likewise, how probable was it that proportionally more of the 8 competitive states (87.5 %) broke in favor of Obama in 2012?

5 Assessment of the probabilistic model

The fact that competitive states are "too close to call" makes it reasonable to model their outcomes as 50–50 for each candidate. From this assumption, it follows that one would expect that about half the competitive states will support each candidate, at least if the results in each of them are independent. But if the votes in competitive states are correlated, extreme results—with most competitive states supporting the same candidate—might be expected.

If a competitive state supports each candidate with probability ½, then the expected number of states that support candidate X is $m/2$, where m is the number of competitive states. In the 2012 election, for example, 7 states voted for Obama and 1 state voted for Romney, so the number of supporters differed from the expectation by $|7 - 4| = 3$. Using the binomial distribution with parameters $m = 8$ and $p = ½$, it is straightforward to calculate how likely such an extreme split is:

$$q = \frac{\binom{8}{0} + \binom{8}{1} + \binom{8}{7} + \binom{8}{8}}{2^8} = \frac{18}{256} \approx 0.0703.$$

Thus, the probability of a split giving either candidate at least seven of the eight states is slightly more than 7 %.[10]

The q-values for the extremeness of the observed splits in the four elections are as follows:

$$2012(7/8 \text{ for Obama}){:}\ q \approx 0.0703;$$

$$2008(4/7 \text{ for Obama}){:}\ q = 1;$$

$$2004(7/12 \text{ for Kerry}){:}\ q \approx 0.7774;$$

[10] In general, we define q to be the probability that the value of the random variable deviates from the mean by at least as much as was actually observed.

$$2000\,(12/16 \text{ for Gore}): q \approx 0.0768.$$

The large q values in 2008 and 2004 are consistent with the hypothesis that the results in the competitive states truly were independent, but the observations for 2012 and 2000 are not. The term "marginally significant" is often used when a q-value is greater than 5 % but less than 10 %. Two such results in four cases suggest that the hypothesis of independence may be violated, but the data do not provide strong enough evidence to conclude significance in the traditional sense (e.g., at the 5 % level).

To carry out a further test, we changed our criterion for competitive states to those in which the popular-vote difference between the two candidates is less than or equal to 3 % (rather than 6 %).[11] Note that in all four elections, the candidate who won more competitive states on the basis of the 6 % criterion also did so on the basis of the 3 % criterion. The results are as follows:

$$2012(2/3 \text{ for Obama}): q = 1;$$

$$2008(3/5 \text{ for Obama}): q = 1;$$

$$2004(4/7 \text{ for Kerry}): q = 1;$$

$$2000\,(6/7 \text{ for Gore}): q = 0.125.$$

Again, we have only a small number of samples, and this test clearly is related to the 6 % test. But because the 3 % test strongly supports the assumption of independence, we conclude that, when combined with the 6 % test, there is no strong evidence against the assumption of independence is violated.

If there is a surprise, it is that the candidates who lost in 2000 (Gore) and 2004 (Kerry) won a majority of the competitive states, based on either the 6 % or the 3 % criterion. In 2000, Gore's substantially better performance than Bush in the competitive states seems attributable, at least in part, to the fact that he won the national popular vote by more than half a million votes.

6 Conclusions

Contrary to conventional wisdom, the noncompetitive states in a US presidential election *do* count, but in a way different from the competitive states. Because it is the competitive states that determine the outcome in most presidential elections, the major-party candidates target, almost exclusively, voters in them.

But the paths to victory of the winners in the last four presidential elections were paved by their leads in the noncompetitive states. The setup power of these states was always substantial—and decisive in one case (2008)—so it is not surprising that the four candidates ahead in the noncompetitive states all won. But it was a close call for George Bush in 2000, because Al Gore's popular-vote victory gave Gore almost enough electoral votes in the competitive states to win.

In 2012, 42 states, plus the District of Columbia, were noncompetitive. They structured the final contest to give Barack Obama a big head start over Mitt Romney. More

[11] The accuracy of state polls has increased since 2000, enabling Nate Silver to predict correctly the election outcomes in all 50 states in 2012 (http://en.wikipedia.org/wiki/Nate_Silver).

specifically, they afforded Obama 4.22 times as many ways to win the election, and rendered Romney's winning coalitions 2.09 times more vulnerable and 4.22 times more fragile than Obama's.

Our assumptions that (i) each competitive state is equally likely to support either candidate, and (ii) their choices are independent, are both consistent with our data in 2004 and 2008, but less so in 2000 and 2012. However, if the 6 % window for competitive states is narrowed to 3 %—which may be appropriate given the greater accuracy of state pre-election polls nowadays—these assumptions are consistent with the election results.

It is true that our calculations are based on information available only after the election. But insofar as state polls before an election mirror the post-election results, these polls foretell which states will be competitive and which will not be, enabling us to estimate the setup power of the noncompetitive states, and therefore the advantage enjoyed by one candidate over the other. By our three component measures of this power, the winner in each of the four elections benefited substantially from his electoral-vote lead in the non-competitive states, which increased his winningness and reduced his vulnerability and fragility.

Our measures describe the situation facing the candidates when the identity of the competitive and noncompetitive states becomes known. We have not attempted to model the responses of the candidates to such information, which would be to allocate their campaign resources to the competitive states in some form of deterministic or probabilistic asymmetric Colonel Blotto game (Roberson 2006). There have been several attempts to use games related to Colonel Blotto to study electoral strategy, including Brams and Davis (1974) and Myerson (1993), but none has specifically addressed the fundamental asymmetry on which we have focused: The noncompetitive states typically give one candidate a substantial advantage.

To conclude, the head start that the noncompetitive states usually give one candidate or the other does matter, even if the voters in these one-sided states cannot change the outcome. In 2012, they gave Barack Obama a big advantage, putting Mitt Romney in a catch-up position which he could not overcome even if he had won as many electoral votes as Obama did in the competitive states.

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